

**Your Signature** \_\_\_\_\_ *This is a closed book exam. Calculators are permitted. There are five problems. Each problem is worth 10 points, for a total of 50 points.*

**Show all your work.** Correct answers with insufficient or incorrect work will not get any credit.

**Score**

1.	(10)	
2.	(10)	
3.	(10)	
4.	(10)	
5.	(10)	
Total.	(50)	

**Sheets attached:** \_\_\_\_\_

1. Suppose that  $n$  people, of which  $k$  are men, are arranged at random in a line. What is the probability that all the men end up standing next to each other?
  
2. An assembler of electric fans uses motors from two sources. Company  $A$  supplies 90% of the motors, and company  $B$  supplies the other 10%. Suppose that 5% of the motors supplied by company  $A$  are defective and that 3% of the motors supplied by company  $B$  are defective. An assembled fan is found to have a defective motor. What is the probability that this motor was supplied by company  $B$  ?
  
3. Solve the following questions and giving **reasons** for your answer.

(a) Let  $X$  be a discrete random variable. Which of the following functions can represent the distribution function  $F$  of  $X$ :-

<p><b>(i)</b></p> $F(x) = \begin{cases} 0 & x \leq -1 \\ 0.6 & -1 < x < 1 \\ 1 & x \geq 1 \end{cases}$	<p><b>(ii)</b></p> $F(x) = \begin{cases} 0 & x < 0 \\ 0.5 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$	<p><b>(iii)</b></p> $F(x) = \begin{cases} 0 & x < 0 \\ 0.4 & 0 \leq x < 1 \\ 0.3 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$
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- (b) Decide whether the following statement is true:- “If  $A$ ,  $B$ , and  $C$  are pairwise independent events then they are independent events.”
  
- (d) Let  $X \stackrel{d}{=} \text{Binomial}(36, \frac{1}{2})$ . Let  $\Phi(t) = \int_{-\infty}^t dt \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}$ . Find the  $r, s, t, u, v$  so that the following approximations are valid:
  - (i)  $P(3 \leq X \leq 20) \approx \Phi(r) - \Phi(s)$
  - (ii)  $P(X = 20) \approx \frac{e^{-t} u^{20}}{v}$

4. Suppose the number of earthquakes ( $X$ ) that occur in a year, anywhere in the world, is a Poisson random variable with mean  $\lambda$ . Suppose that the probability that any given earthquake has magnitude at least 5 on the Richter scale is  $p$ . Let  $B$  be the number of earthquakes in a year of magnitude at least 5. Find the distribution of  $B$ .

5. Let  $X$  be a Geometric ( $\frac{1}{2}$ ) random variable and  $Y$  be an independent Geometric( $\frac{1}{2}$ ) random variable. Let  $W = \max\{X, Y\}$ . Find the distribution of  $W$ .